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EXPLICIT EXPRESSION TO PREDICT THE EROSION
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16. Abstract Using the theory of gasdynamics and heat transfer from a turbulent gas flow to the burning surface of propellant along a permeable wall, an explicit expression is derived to predict the burning rate of the solid propellant with crossflow. Results of the calculation have been compared with experimental data and proved to be correct.			
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EXPLICIT EXPRESSION TO PREDICT THE EROSIVE BURNING RATE OF SOLID PROPELLANTS

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I. Introduction

Erosive burning is an important topic closely related to /84** the design standard of a solid rocket engine. After 30 years of hard work throughout the world, its features are still not fully understood. Representative theories include the L-R equation and K.K. Kuo's chemically reactive turbulent boundary layer theory. In order to clearly understand the law governing erosive burning, the gas flow near the boundary layer of a burning surface is quantitatively analyzed based on existing theory. In addition, an explicit theoretical formula to calculate the erosive burning rate is derived based on macroscopic heat transfer theory.

II. Gasdynamic Analysis of Propellant Surfaces

In order to understand the state of flow of the combustion gas along a surface, the more complicated procedure is to numerically solve a series of equations: the equations of mass, momentum, composition, energy and state of a steady, two-dimensional, chemically reacting turbulent boundary layer. After being simplified, they are as follows:

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$$\partial(\rho u)/\partial x + \partial(\rho v)/\partial y = 0 \quad (2-1)$$

$$\rho u \partial u / \partial x + \rho v \partial u / \partial y = \partial / \partial y [\mu_t \partial u / \partial y - (u \mu_t / \rho c_p) \partial p / \partial x] \quad (2-2)$$

$$\rho u \partial Y / \partial x + \rho v \partial Y / \partial y = \partial / \partial y [(\mu / Sc)_t - \partial Y / \partial y] + \dot{m} \quad (2-3)$$

$$\rho u \partial H / \partial x + \rho v \partial H / \partial y = \partial / \partial y [(\mu / Pr)_t \partial H / \partial y + [\mu_t - (\mu / Pr)_t] \partial / \partial y (u'^2 / 2)] \quad (2-4)$$

$$p = \rho R T / W \quad (2-5)$$

where

$$\rho v^2 = \rho u^2 + \overline{\rho v'^2}, \quad \tau_t = -\overline{\rho u'v'} = \mu_t \partial u / \partial y.$$

Based on the double equation turbulence model, equations for turbulent kinetic energy and turbulent dissipation factor ϵ are used to close the loop for equations (2-1) - (2-5), i.e.,

$$\begin{aligned} \rho u \partial K / \partial x + \rho v \partial K / \partial y &= \partial / \partial y [(\mu + \mu_t / c_1) \partial K / \partial y] \\ &+ \mu_t [(\partial u / \partial y)^2 - (v / \rho c_p) \partial p / \partial x \partial u / \partial y] - \rho \epsilon \end{aligned} \quad (2-6)$$

$$\begin{aligned} \rho u \partial \epsilon / \partial x + \rho v \partial \epsilon / \partial y &= \partial / \partial y [(\mu + \mu_t / c_2) \partial \epsilon / \partial y] + c_{\mu 1} [(\partial u / \partial y)^2 \\ &- (v / \rho c_p) \partial p / \partial x \partial u / \partial y] \epsilon / K - c_{\mu 2} \epsilon^2 / K \end{aligned} \quad (2-7)$$

By choosing the appropriate boundary conditions to solve the above equations, it is possible to find a numerical solution. Using the AP 25% PBAA/EPON 75% propellant as an example, the values of the functions $K = K(y/\delta)$; $\epsilon_t = \epsilon(y/\delta)$ and $\mu/\mu_\infty = \mu(y/\delta)$ are calculated [1]. The calculated turbulent stress follows the following formula: $\tau_t = a(y/\delta) + b(y/\delta)^2$. Here $\tau_t = -1000 \overline{u'v'}/u_\infty^2$, under the flow condition ($u_\infty = 450 \text{ m/s}$; $p = 7.24 \text{ MPa}$; $T_\infty = 2250 \text{ K}$) the constants are $a = 19.9$ and $b = -55.15$. Based on the definition of aerodynamic friction distance:

$y^+ = \tau_t y / \nu = \sqrt{\tau_t} Re_\tau / \sqrt{1000}$, let us choose $y^+ = 30$ as the lower limit of the turbulent region. By substituting it into the equation for y^+ and using the τ_t expression in iterations, it was calculated that $y/\delta = 0.005$. If $\delta = 1 \text{ cm}$ at $y > 50 \mu\text{m}$, it is in the turbulent region. Under practical working conditions, the exothermic reaction layer of the propellant is usually not greater than $200 \mu\text{m}$. Therefore, we can assume

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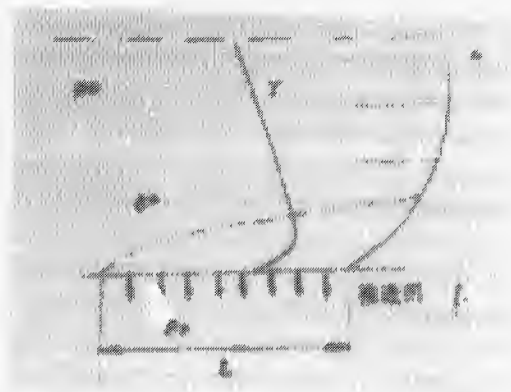


Figure 1. The Flow Model.

1. Propellant.

that gas phase reactions are taking place in the turbulent region.

III. Theoretical Analysis

If the composition of the combustion gas of the propellant sample is different from that of the incident flow, it is possible to assume that there is an interface between these two gases, such as δ^* in Figure 1. Based on the principle of conservation of mass, the following formula can be obtained:

$$\delta^* = \sqrt{\rho_1 r d R T \cdot (1/1.1) g_0 \cdot p_1}$$

Let us choose $l = 1.3 \text{ cm}$, $\rho = 70 \text{ kg/cm}^3$, $u_\infty = 500 \text{ m/s}$.
By calculation $\rho^* = 0.04$. δ^* increases as the value of ρ^* decreases. Under such conditions, the combustion products occupy approximately $400 \mu\text{m}$ above the specimen itself. Based on Planck's principle, the mixing length $l_m = 0.4\delta^* = 160 \mu\text{m}$. This means that at below $400 - 160 = 240 \mu\text{m}$, there should be no disturbance due to the constituents of the incident flow. The heat flow density on the surface is increased only due to the turbulence of the combustion gas of the specimen alone.

We can assume that the burning rate of the propellant is proportional to the heat feedback of gas phase. Under transversal flow (turbulent flow conditions), burning rate can be expressed by the following formula:

$r = Cq = C(-18T/\delta_y + \rho c_p \sqrt{v} T^*)$. If the two terms are separated and expressed by the burning rate, then $r = r_1 + r_2$. The first term r_1 represents the effect of conduction heat conduction, and the second term represents the effect of turbulence on heat transfer. Based on conventional heat transfer theory, it is also possible to express r_2 as follows: $r_2 = Ck(T_1 - T_\infty)$. Based on the principle of conservation of energy, we get $\rho_p c_p r_1 (T_1 - T_\infty) = k(T_1 - T_\infty)$. Here, the proportionality constant C is equivalent to $1/\rho_p c_p$. By using the heat transfer coefficient formula $h = \rho c_p u_\infty St$, we get

$$r_2 = (\rho c_p u_\infty St / \rho_p c_p) (T_1 - T_\infty) / (T_1 - T_\infty) \quad (3-1)$$

Where there is gas coming in along the wall, the expression for the Stanton number is [2]

$$St = St_0 \Phi = C_1 Pr^{-1/2} - C_2 Pr^{-1/2} \Phi / 2 = C_2 Pr^{-1/2} [2/(\Phi^{1/2} + 1)] [1 - (b/b_0)]^2 \quad (3-2)$$

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Where $\varphi = T_1/T_0$, $b = 2\rho_0 r / \rho_\infty C_{D_0}$, b_{cr} represents the value of b corresponding to zero stress on the surface. However,

$C_{D_0} = C_D(Re_0)^{-0.5}$, $\rho_\infty C_{D_0} = C_{D_0}(\bar{\rho}_\infty)$. Thus, we get $b = 2\rho_0 r / C_{D_0}(Re_0)^{-0.5} C_{D_0}(\bar{\rho}_\infty)$ where $\bar{\rho}_\infty = (8/d^2) \int_0^\infty \rho u y dy$. It is also empirically known that $St_0 = 0.023 Re^{-0.5} (Re_0)^{-0.5}$. By substituting St_0 , b , and St into equation (3-1) and then into the formula for burning rate r , we get

$$r = r_0 + A(\bar{\rho}_\infty)^{0.5} d^{-0.5} \{1 - B \rho_0 r d^2 / (\bar{\rho}_\infty)^{0.5}\} \quad (3-3)$$

where

$$A = 0.023 C_{D_0} \rho_0^{-0.5} Pr^{-0.5} (T_1 - T_0) \cdot 4 / \rho_0 (T_1 - T_0) (\varphi^{1/2} + 1)^{-1}$$

$$B = 2 / b_{cr} C_{D_0} C_{D_0} \rho_0^{0.5}.$$

When $r = r_0$ (i.e., $r_e = 0$), based on equation (3-3), /86
the following condition should be satisfied:

$1 - B \rho_0 r d^2 / (\bar{\rho}_\infty)^{0.5} = 0$. Thus, the erosion boundary condition can be obtained: $(\bar{\rho}_\infty)_{cr}^2 = B \rho_0 r d^2$. In order to facilitate design calculation, implicit function in equation (3-3) is made explicit, i.e.,

$$r/r_0 = (\bar{\rho}_\infty)^{0.5} / (\bar{\rho}_\infty)_{cr}^2 \left\{ 1 + (1/2AB\rho_0) \left[1 - \sqrt{1 + 4AB\rho_0 (1 - (\bar{\rho}_\infty)_{cr}^2 / (\bar{\rho}_\infty)^{0.5})} \right] \right\} \quad (3-4)$$

IV. Comparison of Theory to Experiment

Based on equation (3-4) and the specifications for the American propellant J.P.N., we calculated r/r_0 and $(\bar{\rho}_\infty)$

($A = 11.48 \text{ kg} \cdot \text{cm} \cdot \text{s unit}$; $B = 56.6$). The results are shown in Table 1. As compared to the experimental data obtained by Winblast, the error of calculation is not greater than 1 percent when $u < 600 \text{ m/s}$.

Table 1. Results Calculated Based on Equation (3-4).

$u(\text{m/s})$	200	250	300	350	400	450	500	600	700
r/r_0	1.06	1.130	1.22	1.31	1.4	1.49	1.58	1.75	1.93

Symbols

C_{1-4} , C_d , C_p - constants

C_p , C - specific heat for burning gas and propellant

Re_d - ($= \overline{pud}/\mu$)

T_f , T_s , T_j - flame temperature, surface temperature, and initial propellant temperature

Y - mass fraction of burning gas constituent

ρ , ρ_p - density of burning gas and propellant

Subscripts

o - absence of gas blowing onto the surface

' - fluctuation

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